LIMITING CHARACTERISTICS OF INCLINED THERMOSYPHONS AND HEAT PIPES WITH EXCESS HEAT-TRANSFER AGENT

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The authors consider a hydrodynamic model of the limiting heat transfer in inclined thermosyphons and heat pipes under stratified motion of the heat-transfer agent.

The operation of heat pipes with excess heat-transfer agent in a gravity field differs from the method of operation of heat pipes with a nominal charge, in that the return of the liquid heat-transfer agent to the heat supply zone can be accomplished both along the capillary-porous wick and by stream flow of liquid under the action of gravity along the lowest generator of the inside surface. In order to reduce the thermal resistance of heat pipes which use gravity and can be conventionally called gravity-type it is desirable to reduce the thickness of the wick to a minimum, reducing its function to the distribution of liquid over the surface of the heat-carrying zones. In this case the axial limit of a gravity heat pipe will be set by the hydrodynamics of the liquid and vapor flows of heat-transfer agent, interacting on the interface, as was shown in [1] for a pipe position close to horizontal. As a model of such a gravity heat pipe we can take an inclined evaporative thermosyphon with a small amount of heat-transfer agent, which independently warrants investigation. The objective of the present study is to develop techniques for calculating the heat-transfer limit of gravity heat pipes with a wick having low permeability in the axial direction, and of inclined evaporative thermosyphons with a low charge operating in the stratified liquid and vapor flow regime, and also to investigate the thermal resistance of these for near limiting load conditions. The approach used is a development of an idea of [1] for the case of large slope angles; here the following basic assumptions are made: the flow of the liquid and the vapor is stratified, the free surface of the liquid is close to planar (Fig. 1), and the flow regime of the liquid is laminar.

The equations of hydrodynamics for the heat-transfer agent in a heat pipe have the form:

$$\nu_l\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-u\left(\frac{\partial u}{\partial z}\right)_{\text{av}}=\frac{1}{\rho_l}\frac{dP_l^{\text{t}}}{dz}+g\sin\varphi, \tag{1}$$

$$u|_{\omega_1=0}=0,\quad \left.\frac{\partial u}{\partial y}\right|_{\omega_2=0}=-f_v\frac{\rho_v\bar{u}_v^2}{8\mu_l},$$

$$\frac{d}{dz}\left(P_v+\beta_v\rho_v\bar{u}_v^2\right)=-f_v\frac{\rho_v\bar{u}_v^2}{2D_{hv}}, \tag{2}$$

Here Eq. (1) is the boundary-layer equation for a liquid in the Oseen approximation, and Eq. (2) is the equation of vapor motions along a channel of hydraulic diameter D_{hv}. Relations for f_v, β_v allowing for the effects of blowing and suction have been given in [2], but it is hardly appropriate to allow for these effects in the present situation because, first, their influence on the results of the calculation is usually small, and secondly, the shape of the channel section for the vapor is not circular and does not have axial symmetry for the radial component of the flow velocity. Taking this into account, for simplicity we put $\beta_v = 1.33$, $f_v = 64/\text{Re}_v$ for the laminar regime of the vapor, and $\beta_v = 1.03$, $f_v = 0.316/\text{Re}_v^{0.25}$ for the turbulent regime.

We shall write the pressure balance equation for any section of the heat pipe as follows:

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$$\frac{dP_l}{dz} - \frac{dP_v}{dz} = \rho_l g \cos\varphi \left(\frac{dH}{dz} - \operatorname{tg}\varphi \right), \tag{3}$$

where the z coordinate is reckoned from the condenser. It can be seen from Eq. (3) that in the horizontal situation the heat-transfer agent is transported under the action of the hydrostatic head due to the variation of the liquid level along the length. The heat flux transmitted through a given section of the heat pipe is connected with the average velocities of the liquid and the vapor by the explicit relation $Q(z) = \bar{u}_v S_v \rho_v r = \bar{u}_l S_l \rho_l r$. Taking account of what has been said, one can write the mass conservation equation for boundary conditions of the third kind at the outside surface of the heat pipe for known internal heat-transfer coefficients α_c:

$$\frac{dQ(z)}{dz} = -\frac{\alpha_{ex}(T_{ex} - T_v) 2\pi R_{ex}}{1 + \frac{\alpha_{ex}}{\alpha_c}}, \; Q(0) = 0, \; Q(L) = 0.$$

A second boundary condition for Q is necessary to determine the vapor temperature T_v. In the present analysis we restrict attention to the most widespread case of boundary conditions of the second kind with a linear variation of heat flux along the heat-carrying zones and a given T_v:

$$Q(z) = Q_a \frac{z}{L_c}, \; 0 \leqslant z < L_c; \; Q(z) = Q_a, \; L_c \leqslant z \leqslant L_c + L_a;$$

$$Q(z) = Q_a (L - z) \frac{1}{L_e}, \; L_c + L_a < z \leqslant L. \tag{4}$$

For completeness of the heat-pipe description we need an integral relation for the mass of heat-transfer agent charged

$$M = M_s + M_v + M_\delta, \tag{5}$$

where M, M_s, M_v and M_δ are, respectively, the total mass of heat-transfer agent, the mass of liquid in the stream, the vapor mass and the mass of the liquid film on the heat-pipe walls.

We shall briefly describe the method of solving the system (1)-(5). Equation (1) with the appropriate boundary conditions is a two-dimensional boundary problem which it is desirable to consider separately for $\varphi = 0$ in the dimensionless form:

$$\frac{\partial^2 U}{\partial \bar{x}^2} + \frac{\partial^2 U}{\partial \bar{y}^2} - EU = 1, \tag{6}$$

$$U|_{\omega_1=0} = 0, \; \left.\frac{\partial U}{\partial \bar{y}}\right|_{\omega_2=0} = -\frac{f_v \rho_v \bar{u}_v^2}{8\mu_l R \frac{dP_l}{dz}} \equiv K,$$

$$U = \frac{u\mu_l}{R^2 \frac{dP_l}{dz}}, \; E = R^2 \frac{\rho_l}{\mu_l} \left(\frac{\partial u}{\partial z} \right)_{\text{av}}, \; \bar{x} = \frac{x}{R}, \; \bar{y} = \frac{y}{R}.$$

It is convenient to solve Eq. (6) by the Ritz method using R-functions [3]. We shall write normalized expressions for the boundaries of the region occupied by the liquid:

$$\omega_1 = 0.5\,[1 - \bar{x}^2 - (\bar{y} + \cos\Theta_0)^2], \; \omega_2 = \bar{y},$$

$$\omega_3 = \bar{x} + \bar{y} - \sqrt{\bar{x}^2 + \bar{y}^2}, \; \omega = \omega_1 + \omega_3 - \sqrt{\omega_1^2 + \omega_3^2}.$$

The problem of Eq. (6) is solved in the form

$$U_n(\bar{x}, \bar{y}) = \sum_{i+j=0}^{n} C_{ij} X_{ij} + K(\bar{y} - \sqrt{1 - \bar{x}^2} + \cos\Theta_0). \tag{7}$$

The Ritz system for determining the n undetermined coefficients C_{ij} in this case has the form

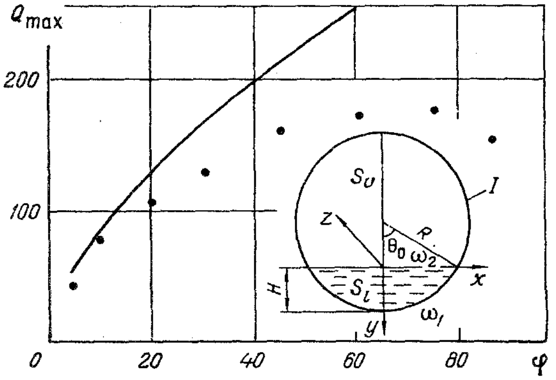

Fig. 1

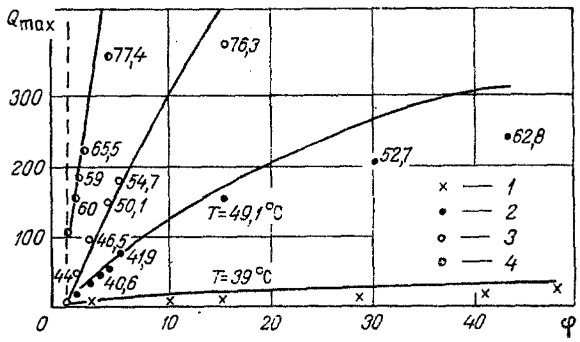

Fig. 2

Fig. 1. The limiting thermosyphon heat transfer Q_{max} (W) as a function of the slope angle φ (deg): the points are the experimental data of [4]; the curve is theory; I) section of the thermosyphon transport zone.

Fig. 2. The limiting heat transfer Q_{max} (W) of a thermosyphon as a function of the slope angle φ (deg), for various charge levels: the points are experimental data, the curves are calculation; 1) M = 0.59 g; 2) 1.38; 3) 2.17; 4) 9.28.

$$\sum_{i,j\geq 0}^{n} C_{ij} \int_{S_l} \left[\frac{\partial X_{ij}}{\partial \bar{x}} \frac{\partial X_{km}}{\partial \bar{x}} + \frac{\partial X_{ij}}{\partial \bar{y}} \frac{\partial X_{km}}{\partial \bar{y}} + E X_{ij} X_{km} \right] d\bar{x} d\bar{y} = \int_{S_l} D_0 X_{km} d\bar{x} d\bar{y} + \int_{\omega_2=0} K X_{km} d\bar{x}\, d\bar{y},$$

where

$$X_{ij} = \Phi_{ij} - \omega \left[\frac{\partial \Phi_{ij}}{\partial \bar{x}} \frac{\partial \omega}{\partial \bar{x}} + \frac{\partial \Phi_{ij}}{\partial \bar{y}} \frac{\partial \omega}{\partial \bar{y}} \right],$$

$$\Phi_{ij} = \left\{ \frac{\bar{y}^2}{2} - \frac{R^2}{2} \left[\cos \left(\operatorname{arctg} \frac{\bar{x}}{\cos \Theta_0} \right) - \cos \Theta_0 \right]^2 \right\} T_{2i}(\bar{x})\, T_{2j}(\bar{y}),$$

$$D_0 = K\left[(1-\bar{x}^2)^{-\frac{3}{2}} - E\left(\bar{y} - \sqrt{1-\bar{x}^2} + \cos\Theta_0\right)\right] - 1,$$

and $T_{2i}(\bar{x})$, $T_{2j}(\bar{y})$ are Chebyshev polynomials of the first kind. The solution of the Ritz system and the determination of the velocity field from Eq. (7) is done with the aid of a computer, and here even for n = 10 we achieve good solution accuracy, and the solution was checked by comparison with the model problems for $\Theta_0=90°$, $E=0$, $\Theta_0\to 0$ and comparison with the results of [1]. The numerical experiments have shown that reverse flows arise for considerable vapor flow velocities on the liquid surface. The pressure gradient in the liquid due to friction and acceleration is expressed as follows:

$$\frac{dP_l}{dz} = \frac{\mu_l}{\rho_l R^4} \frac{Q(z)}{rV}, \text{ where } V = \int_{S_l} U d\bar{x} d\bar{y},$$

or in terms of the friction factor

$$\frac{dP_l}{dz} = -f_l \frac{\rho_l \bar{u}_l^2}{2D_{hl}}. \tag{8}$$

A numerical calculation performed in the present study gives good agreement with the expression for f_l obtained in [1] for E = 0:

$$f_l = \frac{64}{\mathrm{Re}_l} + 8.2 \cdot 10^{-4} f_v \frac{\rho_l}{\rho_v} (2\Theta_0)^6. \tag{9}$$

In the case of very small Θ_0, typical for large heat-pipe slope angles, one can obtain a more accurate expression. We note that for values of Θ_0, tending to zero the ratio of the height of section S_l to its width also tends to zero so that we can reduce the problem to the one-dimensional case, which has an analytical solution. After determining the velocity distribution and integrating over S_l, we obtain the following expression for f_l at small Θ_0:

$$f_l = \frac{(\Theta_0 - \sin\Theta_0\cos\Theta_0)^3}{B\Theta_0}\left\{\frac{12}{\mathrm{Re}_l\,\Theta_0\cos\Theta_0} + \frac{3}{16} f_v \frac{\rho_l}{\rho_v} \frac{\cos^2\Theta_0\,\mathrm{tg}\,\Theta_0 - 2\cos\Theta_0 \ln \mathrm{tg}\left(\frac{\pi}{4}+\frac{\Theta_0}{2}\right) + \Theta_0}{(\pi - \Theta_0 + \sin\Theta_0\cos\Theta_0)^3}\right\}, \quad (10)$$

$$B = -\cos^3\Theta_0\,\mathrm{tg}\,\Theta_0 + 3\cos^2\Theta_0 \ln \mathrm{tg}\left(\frac{\pi}{4}+\frac{\Theta_0}{2}\right) - 3\cos\Theta_0 + \sin\Theta_0.$$

The calculation of Eq. (10) should be made with increased accuracy. We note, for example, that for $\Theta_0=5°$ the difference in the results of calculating the components relating to interaction of the vapor and liquid flows, from Eqs. (9) and (10) is 25%, and for $\Theta_0=20°$ it is less than 5%. For the case of short evaporation and condensation zones at large heat flux levels the value of f_l can be influenced appreciably by blowing and suction of mass from the surface of the stream. Then to determine f_l we use the results of a numerical calculation of the function $V(\Theta_0, K, E)$ but, as is shown by analysis, in most actual cases one achieves good accuracy by putting E = 0 and determining f_l from Eqs. (9) and (10), as has been done in the example calculations below.

Using explicit trigonometric expressions relating the height of the stream region H with its area and the quantity Θ_0, and also expressing the mean vapor and liquid flow velocities in terms of Q(z), from Eqs. (2), (3), and (8) we obtain a nonlinear differential equation of first order for H:

$$\frac{dH}{dz}\left\{1 - \frac{2\beta_v Q^2(z)}{\rho_l\rho_v g\cos\varphi S_v^3 r^2}\left[R\sin\Theta_0 - (R-H)\frac{\cos\Theta_0}{\sin\Theta_0}\right]\right\} =$$

$$= \mathrm{tg}\,\varphi + \frac{1}{\rho_l g\cos\varphi}\left[\frac{-f_l Q^2(z)}{2\rho_l r^2 S_l^2 D_{hl}} - f_v\frac{Q^2(z)}{2\rho_v r^2 S_v^2 D_{hv}} + \frac{2\beta_v Q(z)}{\rho_v r^2 S_v^2}\frac{dQ(z)}{dz}\right]. \quad (11)$$

Equation (11) is solved numerically using a standard Hemming procedure, and the integral condition (5) is implemented by using a target method with the boundary condition $H(L) = H_0$. It should be noted that for large heat-pipe slope angles a solution is reached only by greatly reducing the step size in coordinate z compared with the case $\varphi=0$. The values of M_s, M_v, M_δ were calculated after determining H(z). In the thermosyphon calculation M_δ can be found from modified Nusselt theory, and for large charge levels the value of M_δ is negligibly small compared with M_s. The limit of the heat transfer Q_{max} is defined by the heat-flux region for which the problem of Eqs. (5) and (11) has no solution and is determined by extending H(z) beyond the region of existence [0, 2R]. When Q_{max} is reached, the quantity H(L) in the numerical experiments has a value close to zero; from this one can postulate that in the experiment an increase of the evaporator wall temperature must occur in the region of the end of the heat pipe.

With an increase of the mass of heat-transfer agent charge to some value M_0 the heat-transfer limit increases, and for subsequent increase of the charge mass Q_{max} does not depend on M, and M_0 decreases with increase of φ.

The assumptions made in formulating the problem limit the region of application of the model to the range of variation $0 \leqslant \varphi \leqslant \varphi_1$, where φ_1 is determined by comparison with the experimental data. The use of the model is made a little difficult by the possibility of a boiling regime with vaporization of liquid in the stream; the occurrence of vapor bubbles increases the hydraulic resistance of the stream and forms waves on its surface, which are then subject to the dynamic action of the vapor flow.

For an experimental check of the validity of the chosen model we used a conventional technique to investigate the heat-transfer characteristics of a thermosyphon with a 1Kh18N9T stainless steel body as a function of the charge mass and the slope angle. Heat was supplied by a resistance heater. The surface temperature was measured with the aid of thermocouples stamped into the heat-pipe wall along its lower, side, and upper generators. The thermosyphon had the following parameters: R_v = 5 mm, R_{ex} = 6 mm, L_e = 0.4 m, L_c = 0.22 m, L_a = 0.38 m; the heat-transfer agent was acetone. A comparison of the calculated and measured results is made in Fig. 2. The comparison shows that for better quantitative agreement of the data the hydraulic diameter of the liquid flow in Eq. (8) should be determined from the calculated ratio of the area of region S_l to the total perimeter of the stream; here the error in determining Q_{max} according to the model considered can be evaluated roughly as 30%. Figure 1 compares the calculation with the experimental data of [4] (R_v = 2.5 mm, L_e = L_c =

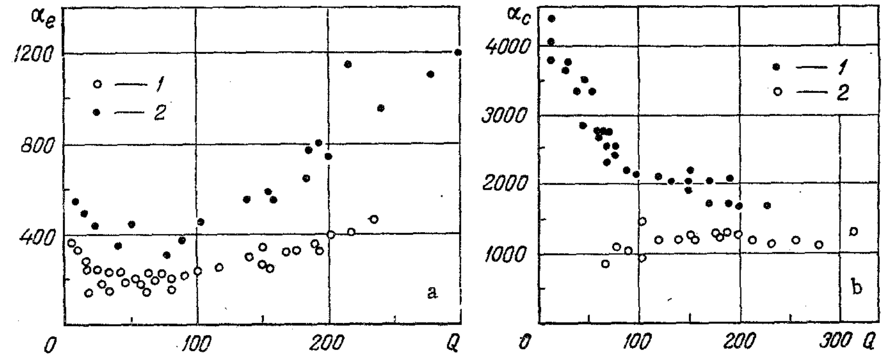

Fig. 3. The heat-transfer coefficients in the heat supply zone (a) and condensation zone (b) of the thermosyphon α_e, α_c ($W/m^2 \cdot deg$) as a function of the power transferred Q (W) at near limiting heat loads: 1) M = 1.38 g; 2) 9.28.

0.16 m, L_a = 0.18 m, M = 3.5 g, T_v = 60°C, heat-transfer agent is water). The calculation error also does not exceed 30% for $\varphi \leqslant 50°$. The experiments have shown, as was hypothesized, that the heat-transfer crisis in the thermosyphon shows up as a sharp increase of the temperature of the end of the evaporator, which is evidence in favor of the hydrodynamic limit of heat transfer, considered in the present study.

It is typical that in the experiment we noted appreciable heating of the upper part of the thermosyphon evaporator relative to the base, for a given cross section, amounting to 30-40°C. This is explained by the fact that only the lower part of the inside surface of the heat pipe, covered by a stream of liquid, is efficiently cooled; the heat supplied is removed from the remaining part by heat conduction of the body and by heat transfer to the vapor by forced convection. The process of heat conduction through the body of the thermosyphon at a section close to the middle of the evpoarator, where the longitudinal temperature gradients are small, is described by the equation

$$\lambda_m \ln \frac{R_{ex}}{R_v} \frac{d^2T}{d\Theta^2} - \overline{T}\alpha_c R_v + \alpha_c R_v T_v + R_{ex} q(\Theta) = 0 \tag{12}$$

with the boundary conditions

$$\left.\frac{d\overline{T}}{d\Theta}\right|_{\Theta=0} = 0, \quad \left.\frac{d\overline{T}}{d\Theta}\right|_{\Theta=\pi} = 0.$$

Here $\overline{T}$ is the wall temperature averaged over the thickness. In view of the fact that $\alpha_c(\Theta)$ and $q(\Theta)$ are functions of arbitrary form, the problem is solved numerically, α_c and q are given in the form of tables for a number of equidistant points with respect to Θ. The heat-conduction equation is approximated, along with the boundary conditions, in a finite-difference mesh with second-order accuracy with respect to Θ, and is solved by a marching method. From the solution of the heat-conduction problem one can approximately evaluate the degradation of the internal heat-transfer coefficients due to drying out of part of the evaporator surface and to blocking of the lower part of the condenser by the liquid. This estimate is relative, since it does not account for possible irrigation of the upper zone of the evaporator by drops of liquid formed with boiling in the stream. In addition, the literature has information of rather low reliability enabling one to calculate heat-transfer coefficients with vapor formation in a liquid stream. As a result of the variation of the coefficients α_c in Eq. (12) for the thermosyphon described and the comparison of the calculated heating values with the experimental, one can conclude that in inclined thermosyphons made of low-conductivity material with a small charge, convective heat transfer between the vapor and the heated inside surface becomes appreciable. In fact, the experimental values of the heat-transfer coefficients α_e, averaged over the evaporator surface and shown in Fig. 3a for a low charge of M = 1.38 g, correspond to the heat-transfer coefficients for convective heat transfer between the wall and the vapor, determined from formulas for established heat transfer, with an error equal to the experimental error. This is explained by the smallness of the area occupied by the stream. With an increase of the charge to 9.28 g the coefficients α_e increase by roughly a factor of two. The thermosyphon slope angle for a given Q can be determined roughly from Fig. 2, since the data presented correspond to nearly limiting levels of charge.

The influence of the charge level on the heat transfer in the condensation zone is inverse compared with the evaporator: with increase of M in the experiments we noted a reduction of α_c, as shown in Fig. 3b. This is explained by blocking of part of the condenser surface by the stream of liquid.

The results presented show that the level of the heat-transfer coefficients in an inclined thermosyphon can be quite low. This touches on a special feature of the heat-supply zone. In cases like these one can reduce the thermal resistance by using a capillary porous coating of small thickness applied to the inside surface.

In conclusion, it should be stressed that the hydrodynamic heat-transfer model examined here allows one to determine the heat-transfer limit of the evaporator of an inclined thermosyphon with a low charge of heat-transfer agent and of a gravity heat pipe with a wick of low transport capability at slope angles of up to 50° relative to the horizontal. The results of the analysis can be used to build a general mathematical model of the gravity heat pipe with excess heat-transfer agent.

NOTATION

u, Velocity; P, pressure; φ, slope angle; z, longitudinal coordinate; r, latent heat of vaporization; λ, thermal conductivity; ρ, density; ν, μ, kinematic and dynamic viscosity; β_v, momentum flux factor; f, friction factor; D_h, hydraulic diameter; S, area; Re, Reynolds number; α, coefficient of heat transfer; $g = 9.81$ m/sec^2; R, radius; Q, heat flux; T, temperature; K, dimensionless shear stress; H, height of the liquid stream; θ_0, semiangle of the stream; V, dimensionless volume flow rate; M, mass of heat-transfer agent; L, heat pipe length. Indices: v, vapor; l, liquid; ex, external; max, maximum; e, evaporator; a, transport zone; s, stream; δ, condensate film; c, condenser; av, average; m, material of body; t, total.

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